Lesson Eighteen: More Complex Shapes

Aims

The aims of this lesson are to help you to:

- remember the formulae to find the area of a triangle, a kite or a trapezium
- find the volume of cylinders and cuboids
- use formulae to find volumes of a range of solids

Why am I studying this?

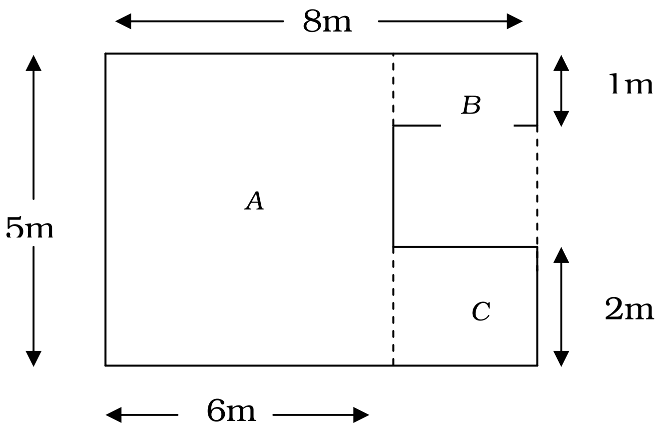
This lesson continues with our work on perimeters, areas and volumes. We have already looked a triangles, circles and rectangles. Here we review less common shapes and extend our range by dealing with more complex ones.

The topics in this lesson are covered in *Key Stage 3 Maths Complete Revision and Practice* pages 110 -115.

Oxford Home Schooling

Area of Composite Shapes

Sometimes the easiest way of finding the area of a shape is to break it into parts. This is particularly useful if you are measuring uneven shapes.



Area of A = 5m × 6m

= $30m^2$

Area of B = 2m × 1m

= $2m^2$

Area of C = 2m × 2m

= $4m^2$

Total area = $30m^2 + 2m^2 + 4m^2$

= $36m^2$

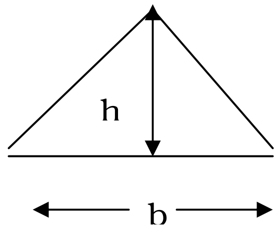
Using Formulae to Find Areas

Many shapes require you to use a formula in order to find their areas. Here are the formulae for some basic 2-dimensional (flat) shapes you are likely to come across. When we first met these, we hadn't yet embarked formally on any algebra, but now this combined revision should be timely and useful for you.

Formula for Finding the Area of a Triangle

Area = $\frac{base \times height}{2}$

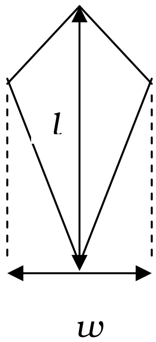
$A = \frac{bh}{2}$



Formula for Finding the Area of a Kite

Area = ½ (width × length)

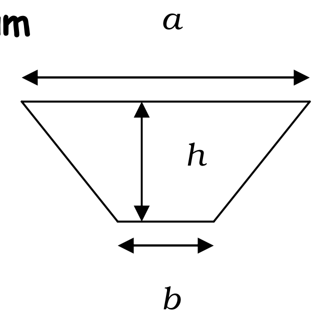
$A = 0.5\,wl = \frac{wl}{2}$



Formula for Finding the Area of a Trapezium

Area = $\frac{height \times (a+b)}{2}$

$A = \frac{h(a+b)}{2}$



Volume

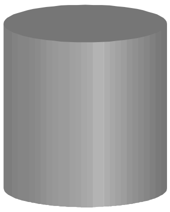
You will remember that area is the amount of space taken up by 2-dimensional (flat) shapes. The amount of space taken up by a 3-dimensional solid shape or a liquid is called **volume.** It is measured in cubic units, so if you are measuring in centimetres you will measure volume in cubic centimetres (cm^3), as happens with such solids as drinks cans and the engine cylinders of motor vehicles.

The formula for the volume of a cuboid (a 3-dimensional 'box' shape with all-right-angled corners, such as a boxed pack of tea-bags) is:

Volume of a cuboid = length × width × height

V = lwh

Volume of a Cylinder



A cylinder is a 3-dimensional circular shape – if you have ever eaten a tube of Pringles you will have come across a cylinder!

The formula for the volume of a cylinder is:

Volume of a cylinder = π × radius × radius × height

$V = \pi r^2 h$

This is a matter of working out the cross-section of the circle (the πr^2 bit) and multiplying by the depth/length of the cylinder. If you think in terms of the round lid of a biscuit barrel, or the head of a piston sliding up and down within the cylinder, this should be simple enough to keep in mind.

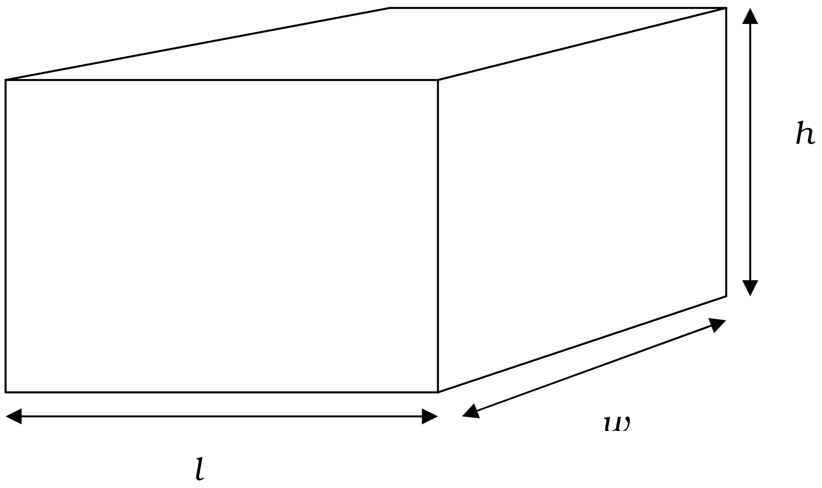
We came across π in the Year 7 course. π is the Greek letter 'pi' (pronounced 'pie'), a 'constant' which equals 3.142 (approx.).

The rationale for it is explained in clearer detail below.

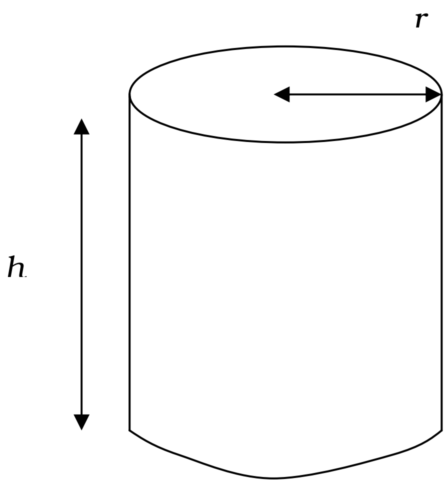
Using Formulae to Find Volumes

When you are finding the volume of a 3-dimensional shape like a cuboid or a cylinder, *i.e.* a prism with parallel sides, you should always be multiplying the area of the base by the height.

$V = lwh$



You find the area of the rectangular base with the use of the formula *lw*, which you then multiply by the height to find the volume.



$V = \pi r^2 h$.

You find the area of the circular base with the use of the formula πr^2, which you then multiply by the height to find the volume.

Tips for remembering how to calculate Area and Volume

- Area units are squared, *e.g.* 5cm^2 (*i.e.* in two dimensions)
- Volume units are cubed, *e.g.* 7cm^3 (*i.e.* in three dimensions)
- Use the same units for each measurement – measure height, width, length and radius using the same units
- $1000\ \text{cm}^3$ = 1 litre (= 10 by 10 by 10 cm; compare this with the actual dimensions of a 'Tetra-brik®' such as you might have in your kitchen with fruit juice, or similar)

What follows is a set of reminder activities, to refresh your skills with the Shape topic and a bit of Algebra (including substituting values into a formula)

Activity 1

A rectangular patch of lawn measures 9m long by 6.5m wide.

(a) What is the area of the lawn?

(b) A tree grows up through the lawn, with a ring-shaped seat around it. The diameter of the seat from edge to edge is 1.9m. What circular area of lawn is covered by the tree-trunk and seat together?

Activity 2

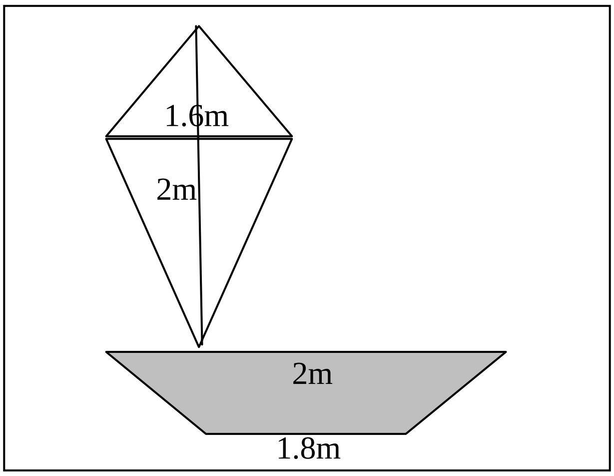
A cuboidal packet of sugar cubes measures 7cm by 4cm by 5cm.

(a) How many cubes are in the box, assuming they are $1cm^3$ each?

(b) The pack is made from thin sheet card. Like any cuboid, it has three matching pairs of faces: a top and base, left and right sides, a back and a front. What total area of packaging is there in one such cuboid? (You can ignore any flaps and overlaps.)

Activity 3

A seaside church is looking to put an eye-catching new logo on the wall of its building, to the following design:



The upright 'mast' of the cross stands life-size at 2 metres, and its 'arm' beam measures 1.6 metres from tip to tip.

(a) What will be the area of the kite-shaped 'sail' panel behind the cross?

The 'boat' portion of the logo is in the form of a trapezium. Its upper parallel edge is again 2m and the lower edge is 1.8 metres long. The boat is 0.6 metres 'deep'.

(b) What will be the surface are of the 'boat' panel?

Activity 4

Mandy bakes a cake in a tin whose diameter is 8 inches.

(a) What is the area of the base of the tin?

The cake rises 3 inches up the side of the tin.

(b) Assuming the cake is flat-topped (thus making a cylinder), what is the volume of the cake?

A friend of Mandy's wants a cake to the same recipe for her child's birthday – but, for decoration purposes, it needs to be 'square' (or, more strictly, cuboidal – but baked in a tin with square cross-section).

(c) Working from your answer to (a), can you think of a standard size of square tin (numbered in whole inches) that would come close enough to having an equal 'footprint' with the original round cake? (The aim is to keep the 'depth' of cake more or less the same, so we can ignore it for this purpose and concentrate on getting a good match for the base area.)

Activity 5

You have probably been using a pencil to sketch and calculate the previous questions. Now put the pencil down and do this final question purely by arithmetic!

(a) Estimate the length of the pencil in suitable units, and check this with a ruler. You can ignore the complicated conical-cum-hexagonal shape at the sharp end.

(b) Estimate the width across the 'non-business' end of the pencil, on a similar basis. Assuming you have a standard pencil which is hexagonal in cross-section, you should measure this between the maximally opposite corners, rather than from 'flat to flat'.

(c) Similarly, estimate and then check the short measurement across any one 'side' of the pencil, such as where any lettering might be.

(d) Using these data, try and work out what volume of wood is in the body of the pencil. Your answers to (b) and (c) should enable you to split the overall hexagonal cross-section into two matching trapezia, which should help considerably.

(e) Estimate, then check, the diameter of the 'lead' in the pencil; and from this, work out the total volume of the lead (a long, thin cylinder of course).

(f) Finally subtract (e) from (d) to discover how much of your pencil consists of wood.

Suggested Answers to Activities

Activity One

(a) $9 \times 6.5 = 58.5$ m^2

(b) Circle has diameter 1.9m, hence radius = 0.95m

Area of circle = $\pi r^2 = \pi \times 0.95 \times 0.95 = 2.835$m^2. We have taken pi and 'chipped away at it' twice-over, multiplying it by just under 1 each time, so an outcome of a little less than 3 seems fully reasonable. $\pi \times 0.95$ (the first time) = 2.985 and we have gone down again from there.

Activity Two

(a) $7 \times 4 \times 5 = 140$ lumps

(b) Work out one of each single separate side type, add them up & double:
1 top at $7 \times 5 = 35$
1 side at $4 \times 5 = 20$
1 front at $7 \times 4 = 28$

Subtotal for one of each = 35 + 20 + 28 = 83
Double for whole package = $2 \times 83 = 166$ cm^2

Activity Three

(a) Area of kite = half of (height × width)
Much the simplest is to multiply 1.6 by 'half of 2', leaving <u>1.6m^2</u>

(b) Top of 'boat' = 2m, bottom = 1.8m, so 'middle' = 1.9m
Depth is 0.6m, total area of boat shape = 1.9×0.6 = <u>1.14m^2</u>

Activity Four

(a) Diameter 8", so radius = 4"

(b) $\pi \times 4^2 = 16\pi = 50.265$ sq.in. (say 50 sq.in. to the nearest whole)
$50 \times 3 = 150$ cubic inches

(c) The closest square to 50 is 49 (= 7^2), and indeed a 7" × 7" tin once did the very job successfully in real life!

Activity Five

This will of course vary immensely depending on what kind of pencil you have, and how far used it happens to be. We will work with illustrative values to suggest how such a question might realistically work out.

A standard pencil would be unlikely to be much longer than 15cm when fresh from the factory. Let's roughly halve this and go for an arbitrary 8cm. If we halved that once more to 4cm it would be too short to work with comfortably.

(a) The long diagonal is unlikely to be as much as 1cm; 7mm would be typical, as checked by the author.

(b) Measuring one side is fiddly but it's obviously going to be a handful of mm, if that. The author's same pencil had a measurement of 4mm.

(c) One half of the pencil would have a cross-section in the form of a trapezium with its long ('inner') side of 7mm and its parallel outer side 4mm. The one other measure we need is the vertical height of this trapezium. If the 'flat height' of the pencil is 6mm, half of this would clearly be 3mm (its 'flat radius', so to speak).
So the trapezium is 3mm high, with a measure across its middle that is halfway between 4mm and 7mm, *i.e.* 5.5mm.
$3\text{mm} \times 5.5\text{mm} = 16.5\text{mm}^2$
… and we double this, as there are 2 trapezia facing each other:
$2 \times 16.5\text{mm}^2 = 33\text{mm}^2$
Then we multiply by the length of the pencil (8cm, which we re-express as 80mm to keep matching units throughout):
$80\text{mm} \times 33\text{mm}^2 = 2{,}640\ \text{mm}^3$
Rounding this arbitrary outcome slightly to 2,500 mm^3 (perhaps the true volume when next the pencil is shortened slightly, e.g. by sharpening), we can 'bundle' four such pencils into 10,000 mm^3 which is 10 cubic centimetres. A hundred times as many as this would fit into a litre (= 1,000cc). If ever you've seen a bulk pack (e.g. in school), you can probably picture a block of hexagonal pencils crammed neatly together into a volume that compares in size with a litre Tetrabrik® of fruit juice, and containing several dozen pencils at least. 400 in a litre would be equivalent to 20×20 pencils, each 10cm long.

(d) The lead diameter is most likely 2mm. 1mm would have been quite ungenerous; 3mm would have been nearly half the barrel; neither of those seems realistic. If you look end-on at the pencil when it's sat flat, the lead does seem to occupy the middle third of its cross-sectional height, so 2mm out of the 6 is probably a fair value.

This also gives us a nice easy radius of 1mm, which squares to 1mm^2, so the cross-section of the lead is $1^2 \times \pi \text{ mm}^2 = \pi \text{ mm}^2$ (like our old bike-wheel, only this time in miniature).

We multiply this by the length of the lead (80mm again) to give
$80\,\pi = 250\text{mm}^3$ or so (3 × 80 would have been 240)

(e) $2{,}640 \text{ mm}^3 - 250\text{mm}^3 = 2{,}390\text{mm}^3$