Lesson Seven: Perimeter, Area and Volume

Aims

The aims of this lesson are to help you to:

- measure the perimeter and area of 2-dimensional shapes
- learn about π (pi)
- calculate the area and circumference of a circle
- calculate volumes

Why am I studying this?

This lesson contains quite a lot of work using formulae. Read it carefully and go back to anything you find difficult. Remember you can always talk to your tutor about anything you don't understand!

Oxford Home Schooling

Perimeter

You can see that the rectangle above has certain measurements – it is 6 cm long and 3 cm wide. Using those measurements you can calculate the distance all the way around the shape. This distance is called the shape's **perimeter** (from two old Greek words: *peri* meaning 'round', in the sense of a periscope for looking around, plus *meter* in the sense of 'measurement'; hence meaning 'round-measurement', which is exactly what it is!) There is quite a bit of ancient Greek in the study of shapes (*geometry* as they & we call it: literally, 'for measuring the world'). They were one of the earliest great mathematical peoples, who indeed laid the 'groundwork' of this vital subject.

To work out the perimeter of the rectangle above, you can either add each of the sides together, one by one:

Example

3cm + 6cm + 3cm + 6cm = 18cm

Or, because you know that there will be two sides measuring 3cm and two sides measuring 6cm, you could calculate:

Example $2 \times (3\text{cm} + 6\text{cm}) = 18$ cm

If you have ever tackled any algebra, you may be aware that you could write the equation above as a formula:

Example $P = 2(l + w)$

The perimeter = 2 × (length + width)

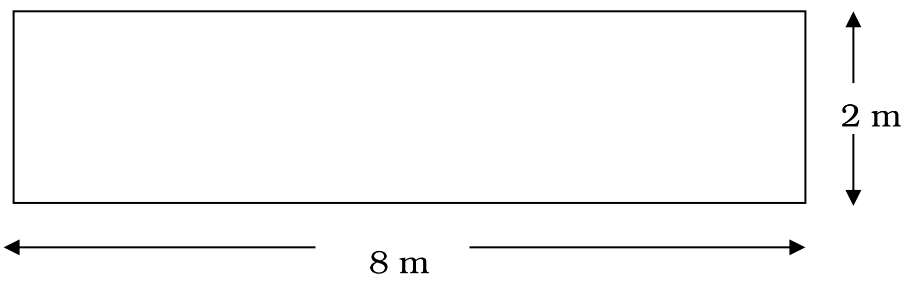
If this notation appears strange at first, you will probably get used to it quite quickly. Think what it means (remember our recent definition of a rectangle as 2 pairs of sides: one long and one short pair), and the sense of it – the *elegance*, even – should be reasonably clear!

Area

The amount of surface space covered by a flat shape is called its **area**. Note that area is different from perimeter – the perimeter of the rectangle above was the distance *around* it, whereas the area of a shape is the entire space that it *covers.*

We measure area in **square units**. If we are calculating the area of a shape with a length and width in centimetres then its area would be in square centimetres (cm^2). If we were calculating with metres then the answer would be given in square metres (m^2).

The area of a rectangle is calculated by multiplying the length of the shape by the width of the shape.



This rectangle has an area of sixteen square metres ($16\ m^2$). Its perimeter is twenty metres (20 m).

Just as you did with the perimeter of a shape, you can use algebra to help you remember which sums you need to do:

Try to memorise the formula for the area of a rectangle:

$$A = l \times w \qquad \text{or} \qquad A = lw$$

Area = length × width

Activity 1

Try the following questions involving rectangles.
You should not need to use a calculator during this Activity, but should show your workings.

1. What would be the perimeter (P) and area (A) of the rectangles with sides as follows?

   (a) 6 and 8 cm
   (b) 7 and 13 cm
   (c) 11 and 12 metres
   (d) 18 millimetres wide by 24 mm tall (a postage stamp)
   (e) A sheet of standard A4 paper, 210 × 297 mm
   (f) 21 yib* long by 9 yib wide
   (g) A football pitch, 105 × 68 metres

   * As you may know, the *yib* is the unit of measurement on the planet Xarp.

2. An ornamental border strip is to be run right round the edge of a notice-board. The board is 3 metres long by 1½ metres high. What length of strip will be needed?

3. A tasseled fringe is sewn round the edge of a rectangular canopy. The canopy measures 3 metres by 2 metres.

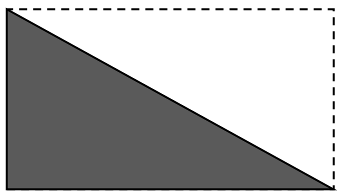
   (a) What is the length of the perimeter of the canopy?
   (b) If the cord in the tassels hangs down by 5cm in each little loop (and back up again, don't forget), and there are two such loops hanging from each 1cm round the rim of the canopy, how much cord altogether will be needed to make the fringe?

Triangles

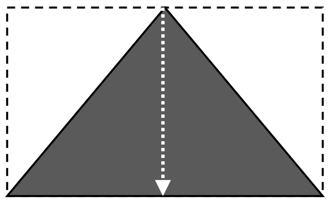
So much for shapes with 'nice right-angled corners' (we should still be calling them *vertices*, of course ...), which are simple enough to calculate once you're used to them.

The second group of **plane** (= flat) straight-sided shapes are those with sides that don't always meet at right-angles. This includes triangles, parallelograms and their kin which you met in another recent lesson. It's those slanting sides that are the problem ... until you understand how to deal with them.

Let's take a simple triangle in a situation such as a signal-flag:

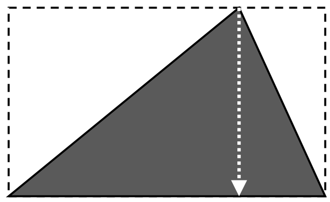


Clearly it occupies half the rectangle, so if the dimensions of the rectangle were 20 and 30cm, the area of the rectangle would be $20 \times 30 = 600\ cm^2$, and the area of the triangle would be $300\ cm^2$ which is half as much.



If another triangle – *any* other triangle – sits in the box, with its base sharing all of the base of the box and its **apex** (its 'point') just touching the top of the box at any one spot, the triangle will always be occupying half the area of the box.

If you're not sure about this, picture it as a medium-close-up photo of a traditional camping tent. Run another line down from the ridge to the base (shown here as a dotted white, zip-style line) – and you have effectively divided the overall shape into two matching halves. Of each side, the tent occupies exactly one half and the rest is fresh air. In total, the 'tent' covers half of the oblong photo frame.



The equivalent is still true even if the tent is **asymmetrical**, *i.e.* not dead-centre. It still occupies half of the left-hand side (the same old proportion, though of a bigger actual area), and half of the right, and hence half of the whole frame just as before.

All of which leads us to a surprisingly simple formula for the area of a triangle ...

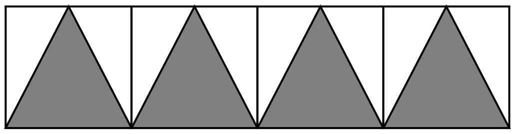
If you know the length of its base and its **perpendicular** height (*i.e.* vertical, meeting the base at right-angles) you can calculate the rectangular area of the 'frame' and then simply halve that. Just make sure you don't try using the lengths of any *slanting* sides in your calculations, because for this purpose those are *irrelevant* and *treacherous*.

To put it even more succinctly:

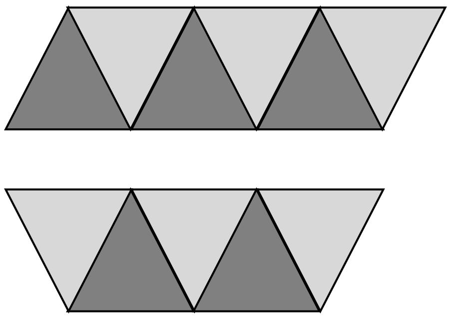
Area of triangle = (base × height) divided by 2

Or indeed, $A = \frac{bh}{2}$

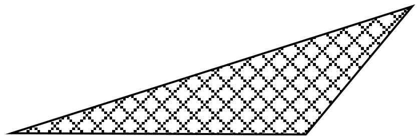
The makers of Toblerone® chocolate bars are smart enough to market it in a distinctive shape: triangular in **cross-section**, so as to remind people of the Alp mountains while also, literally, standing out from the usual 'flat-bed', tiled-rectangle slab bars made by most other brands. But imagine, for a moment, that they were then dumb enough to package it in 'toothpaste-style' (*i.e.* square-cross-sectional) cardboard slips, purely so as to stack them more easily into their trucks, which seen end-on would look like this ...



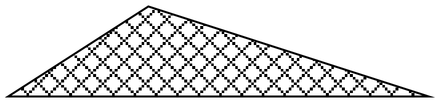
... you will now appreciate that fully <u>*half*</u> of the inside of each truck would be occupied by fresh air, instead of crammed full of chocolate to sell! As you may know, **equilateral** triangles (*i.e.* ones where the sides are <u>*all*</u> of identical length) can in fact be stacked twice as efficiently as that, leaving no gaps at all:



The formula we've discovered here will work for any triangle – equilateral, isosceles or **scalene** (a common-or-garden 'mongrel' triangle with no special features or symmetries) – provided we calculate it with the apex somewhere perpendicularly above the base. If you get an overhanging 'tow-truck jib' triangle like this...



... you'd be better flipping it over onto its longest side and try to establish the ('new') vertical height.



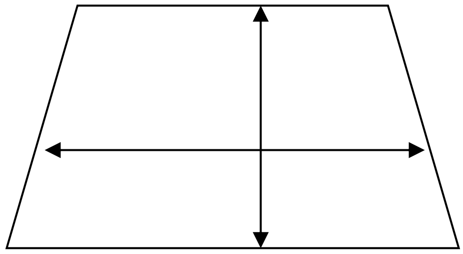
Other 'slant-sided' shapes

Most other shapes with straight-but-sloping sides use a similar formula.

For our old friend the Parallelogram, we slice off one end with a vertical 'cut' and tuck the spare piece into the underhang at the other end (in this case, the paler grey 'stump' moves back to where the white triangle is shown) so as to make a neat rectangle of equivalent area. We then use the plain 'base-times-vertical-height' formula ($A = bh$) to work out the area of the rectangular version.

The same obviously goes for a Rhombus.

When we meet a Trapezium, the technique is a little different but founded on similar principles:



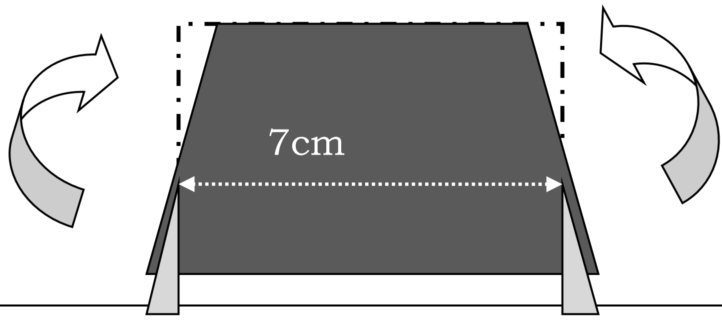
We need the vertical height of course, but also some measure of how *wide* the trapezium is: tricky at first glance, isn't it, with those slipaway slanting sides? How do we get a reliable width or 'waistline' measurement?

Quite simple, actually. We add the lengths of the two parallel sides (usually shown as top & bottom, and maybe also with parallel-line indicator marks to remove any possible doubt); then we halve that total. Let's say that in this example, the lengths are 8cm and 6cm:

$8 + 6 = 14$
$14 \div 2 = 7$

Obviously enough, 7 lies halfway between 8 and 6, so that would be the 'waistline' measurement here. We then multiply that by the vertical height (in this case maybe 5cm) to give a total area of 35 cm^2.

If you prefer to think of this more graphically, in terms of 'chopping & filling', this diagram may help: it uses the same shading scheme as our earlier one for 'curing' the parallelogram. You might also prefer to picture it as 'tucking up the flaps'.

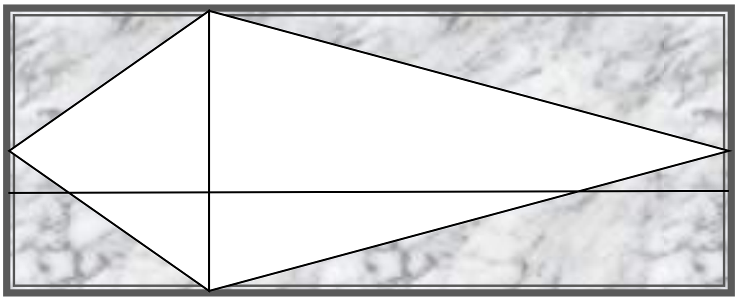


It doesn't even matter if the trapezium is **scalene** (*i.e.* with lopsided or otherwise un-equal diagonals – like those little ramps you see in shoe-shops, or for crocking-up cars for repairs underneath). The same principle still holds good:

$$\frac{(\text{Base + parallel top})}{2} \times \text{vertical height}$$

The last of our slippery straight-sided shapes is the Kite. In fact it brings us back rather tidily to the common principle for all such shapes, *i.e.* that a triangle (in certain defined, relevant and usually easily-spotted circumstances) is half the area of a rectangle that neatly encloses it.

Here's a kite fresh from the toyshop to show what this means:



If you take *<u>any</u>* one (or indeed a next-door pair) of the 'sub-rectangles', you'll find the kite covers exactly half of it. Hence we can work out the area of the entire kite by halving the area of the box (whatever that happens to be), and all done without a moment's faffing-about over the length of any particular slanting sides! Surprisingly easy isn't it, once you know how?

A handy little hint

While working with triangles and the like, if you need to halve a 'rectangular total', you might find it smarter just to halve one of the numbers before going into the rectangle calculation – particularly if either of those figures is even, and/or high.

For instance, the area of a triangle covering half of an oblong flag, where the flag's dimensions were 25 by 40 cm, would be

$$\frac{25 \times 40}{2} = \frac{1000}{2} = 500 \text{ cm}^2$$

… but it might have been simpler to think '25 times half-of-40 = 25 x 20 = 500'). Just remember you only need to do the halving *once* altogether. You can use this where appropriate in the following Activity, if you find it helpful.

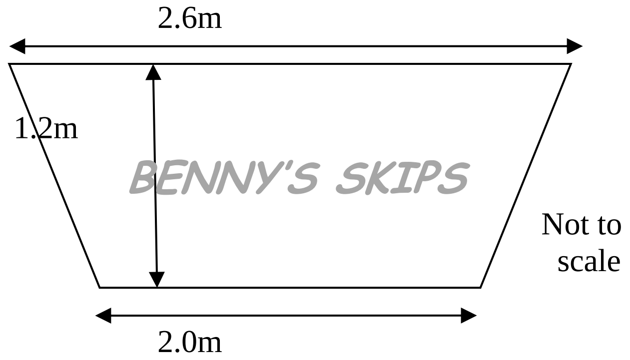
Activity 2

A selection of questions on 'Order 2' polygons:

1. The 'business end' of a triangular road warning sign is 50cm high and 50cm across the base. What is its surface area?

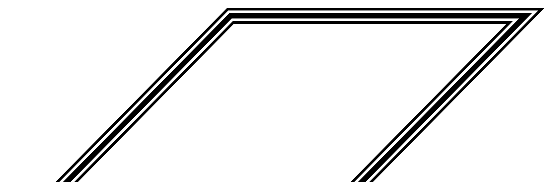
2. Sam's new kite (a traditional one, shaped like the type we've been studying) arrives in a big box 100cm long by 75cm wide. Assuming the thickness of the box to be negligible, and that all four corners of the kite are touching the inside of It, what should Sam expect the total 'sail area' of the kite to be?

3. Seen sideways-on, a builders' skip has a base 2m long; its upper, open lip runs parallel to the base, 1.2m above ground level, and is 2.6m long.



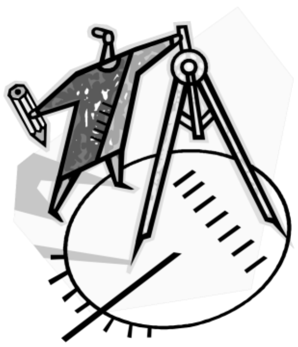
(a) What would be the surface area of <u>one</u> side of the skip?
(b) What would be the combined area of <u>both</u> sides?
(c) Assume that the skip Is 1.5 metres wide. What sould be the area of the bottom panel of the skip?
(d) If the sloping ends of the skip are each 1.3 metres 'long', how much area of metal is needed to make <u>each</u> of them?
(e) What is the total area of sheet-metal needed to make a skip, assuming no doubling-up or loss where joins are made?

4. An emblem in the shape of a rhombus is to be made, to go on the front of a building. Each side measurement of the rhombus is 80cm.



The vertical distance between parallel sides is 60cm.
(a) What is the surface area of the emblem?
(b) If a single, bendy neon light-tube is to run round the edge of the emblem (ignoring any complications for overlaps or corners), what length of light tube will be needed?

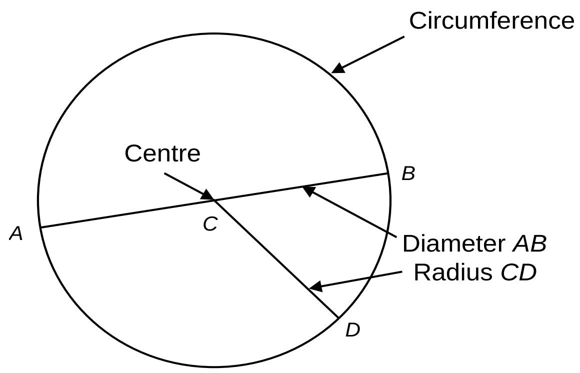
Circles



We now move away from straight-sided shapes altogether and into 'Group 3', the magical world of the circle: such a simple, elegant yet fascinating shape, even when we see dozens of them every day.

The circle is very important in Maths, and if you are hoping to study GCSE Maths you should be prepared to learn many ways of calculating with circles.

You will need an adequate pair of compasses for lessons involving circle work. The diagram below shows some of the basic parts of a circle. You should learn these off by heart.



The **diameter** of a circle is the straight line from one side of the circle to another, passing through the centre. It is the width of the circle.

The **radius** of a circle is the straight line from the centre of the circle to any point on the edge; it is always half the length of the diameter. So it is half the width of a circle.

The perimeter of the circle is what we call its **circumference** – it is the distance all the way round the circle. We only use this word in relation to circles, instead of calling it a 'perimeter'.

Now it's time for a little practical experiment. You need to fetch a dry mug or glass, such as you might otherwise be sipping from as you work; you also need a 30cm ruler, or, better still, a flexible tape-measure (as in a sewing kit, or possibly a tool-box).

As best you can, measure the diameter edge-to-edge across the rim of your mug: you'll most probably get a value of 7 or 8 cm. If you have unusual mugs, don't worry! Try this same investigation with whatever values you do get.

Then measure round the circumference of the rim. With a flexible measure this should be relatively easy; if you're rolling a ruler round it, start with 'zero' at the handle and do your best not to slip. If you run out of 30cm ruler on your way round, you must be dealing with an outsize coffee-cup!

The chances are that your readings will be somewhere in the range of:

Diameter 7cm, circumference 22cm

Diameter 8cm, circumference 25cm

Diameter 9cm, circumference 28cm

(these being rounded values, near enough for present purposes).

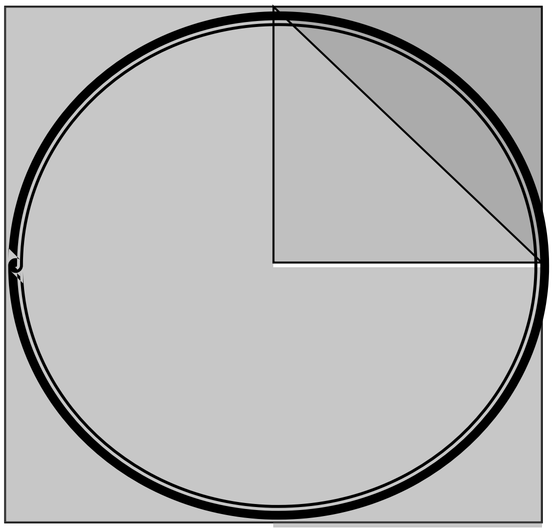
Let's take the middle one as being fairly representative. You will be aware that 3 × 8 = 24, so it seems, at first glance, as though the circumference is 'three-and-a-bit' times the diameter. Similarly, 3 × 7 would have been 21, or 3 × 9 would have been 27. There seems to be some general connection, doesn't there?

Somewhere in the Old Testament of the Bible, where it's cataloguing King Solomon's lavish equipment for the First Temple, a vast ceremonial dish or basin is described as measuring 3 units round and 1 unit across. This 3-to-1 **ratio** of circumference to diameter is quite handy for estimating purposes (and hence for checking calculations later on) but it's not absolutely accurate.

Before we reveal the secret – such as it is – let's apply a similar approach to area rather than length.

Imagine that (for reasons we needn't go into) you need a new bicycle wheel. For ease of calculation we'll assume the diameter of the wheel-rim is 24 inches, *i.e.* that the radius is 12 inches, otherwise known as 1 foot.

Your brand new wheel is delivered, by courier, in a box, much as though it were an outsize pizza:



You fold down one corner of the box just to check *(see top right)*, and there's the wheel peeping out.

Now, thinking mathematically, let's look more closely at that corner (a quarter of the whole box, as it happens). Obviously the wheel covers all of the 'central' section, underneath where the triangular flap's been folded back. But, being a wheel, it also bulges outwards towards the upper right-hand corner of the box. In fact, it seems fair to guess that the wheel covers about half of the darker triangle in that top corner, more or less; in other words, about ¾ of that whole upper-right 'quadrant' of the box, and presumably, therefore, about ¾ of the entire thing all the way round.

Provisionally then, the area covered by the wheel would be ¾ of 4 square feet, because the box is 2 x 2 = 4 square feet, as we saw. This circle has an area pretty close to 3 square feet.

Enter *Pi*

'Pretty close to 3', eh … ? Didn't we discover a rather similar value a little earlier?

Well – yes, we did, but unsurprisingly, those ancient Greeks beat us to it by a several centuries, and they called it *pi*, which was their equivalent for the letter *p* (as in *perimeter*, no doubt):

$$\pi$$

We usually pronounce π identically with 'pie', as in 'shepherd's'.

No matter what the size of the circle – it could be the Earth's equator, or something even huger – if you divide its circumference by its diameter, your answer should always be just over 3. This number can be expressed more accurately as 3.14 (2 decimal places) or 3.142 (3 decimal places); and its usual handy value in non-decimal form is reckoned as 22/7, bringing us back towards the range of values for the mouth of your mug earlier on.

Activity 3	See if you can find a key on your calculator for π. Write down the number that shows on the screen when you press the pi key.
✎	You should see 3.14159265…

Pi is an infinite number (a number that never ends), but don't worry! You needn't learn off all the decimal places that your calculator will have given. But it *is* very important that you learn pi correct to two or three decimal places, so that you can calculate with it easily.

Finding the Circumference of a Circle

To find the perimeter of most straight-sided shapes is fairly simple, as you can see from the earlier part of this lesson. Finding the *circumference* of a circle is different, however, as you can't accurately use a ruler to measure it in the way that you can with any other shape.

So to find the circumference of a circle you need to calculate, working from measurements that you can take, which brings us back to straight lines: the lengths of the circle's radius and diameter, which you can measure with a ruler. There's a bit of a knack to measuring a diameter unless the centre is shown for your reference, but with a little experience you should find circles aren't as 'slippery' as you might fear. And of course, whatever their size, they are all exactly the same shape!

Then you use these formulae in your calculations:

Circumference = π × diameter $\quad C = \pi d$

Circumference = 2 × π × radius $\quad C = 2\pi r$

Finding the Area of a Circle

Finding the area of a circle requires a formula as well, working accurately in reverse of our 'bike wheel' observation.

In that case, the box was 2ft in each flat dimension, making its area 2 x 2 = 4 ft^2. Back then we reckoned the area of the 'incircle' at about ¾ of the total, *i.e.* ¾ of 4, which of course is 3. In fact, the true value could be reached using not 'three-over-four', but '*pi*-over-four' (giving us, in fact, π ft^2 which is '3-&-a-bit').

The official formula is:

Area of a circle = π × radius × radius

Or in letters: $A = \pi r^2$

In the case of the bike wheel, the radius was 1 ft, and 1 when squared remains 1, so in effect that number just 'dissolved' into the calculation. Far more often the radius won't be one single unit's-worth of anything; the radius of your mug was probably about 4cm, for instance, so if it came in a square-based box, the box lid would have been 8 x 8 = 64cm^2 or thereabouts (assuming the mug handle doesn't stick out very far!).

Let's try another example altogether.

Example

Find the area of a circle with a radius of 8cm.

$A = \pi r^2$

$A = \pi \times r \times r$

$A = \pi \times 8\text{cm} \times 8\text{cm}$

$A = 3.14 \times 64\text{cm}^2$

$A = 200.96\ \text{cm}^2$

Activity 4	To 'round off with' (ho, ho …) here are some circle-based problems.

1. A circular duck-pond has a diameter of 5 metres. How far is it round the edge? Express your answer to a sensible, everyday degree of accuracy.

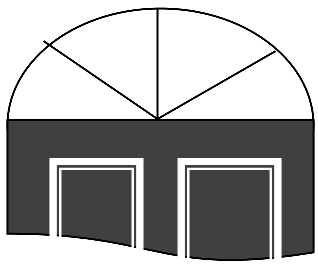
2. (a) The world is not in fact a perfect globe shape; but if it were, and if you take Its equatorial radius as 6378km, how long ought the whole Equator to be?
(b) The Earth's radius at the Poles is said to be 6357km. If you based a 'great circle' on that figure (*i.e.* a transit from North to South Pole, passing at a 'flat' perpendicular through the Equator, and back up the other side to the North Pole), how long a journey would that be?

3. I am not allowed to walk straight across the large round cricket-field on our village green, but I know that I can go right round the outside of it in about two-and-a-half minutes (150 seconds). If I went straight across the middle at the same pace, how long would it take me to reach the far boundary?

4. A goat is tethered to a tree in the middle of a field, on a rope 4 metres long. Ignoring any area taken by the tree itself and its roots, on what total area of grass can the goat graze?

5 A long 'boom' of floats Is put out from a boat, to stop oil escaping beyond a circular patch of sea. The maximum length of the boom is 2,500 metres. Assuming it floats out to form a near-perfect circle, what area of sea surface will be protected within it?

6. Mrs Ippey's cottage has a semi-circular 'fanlight' window over Its front door:

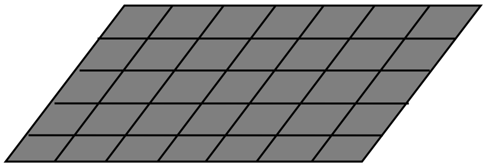


The diameter of the fanlight is 90cm.
(a) What is the total area of glass in the fanlight as a whole? (You can ignore the radial wooden struts for this purpose; and remember that it's only <u>*half*</u> a circle!)
(b) Mrs Ippey decides to seal up the fanlight and put in a new door containing a rectangular frosted window (of about the same area as the fanlight, in order to let in similar levels of light). What sensible dimensions can you think of for such a window? You should probably be reckoning on pairs of side-lengths, to the nearest 'round' 5 or 10cm, giving a value in cm^2 reasonably close to your answer for (a).

Volume

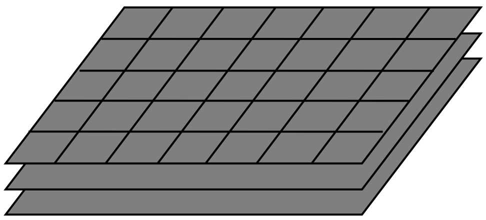
So far we've dealt with **plane** (*i.e.* flat) 2-dimensional shapes, in terms of their length and width. Of course, there is a third **dimension** – typically, 'height' off the page.

Let's go back to our 'chocolate' theme for a moment, and to an ordinary flat-bed tablet of it, such as this:



It has 7 × 5 'squares' = 35 portions in a single layer. For present purposes we're ignoring the actual physical height of a 'square', which would probably be a handful of millimetres.

3 such tablets in a multi-buy would fairly obviously give you 3 times as many 'squares', *i.e.* 105 of them.



We might express this as '7 × 5 × 3 = 105', where the 7 represents crosswise length, 5 marks the breadth front-to-back and 3 stands for the height, in terms of how many layers. These are the three **dimensions** of the overall wadge of chocolate, and their total actual value would be expressed in **cubic** units such as cm^3 ('cubic centimetres').

One single cube of anything (a sugar-cube, say) is obviously in a layer all by itself with nothing beside or behind it, and

$$1 \times 1 \times 1 = 1$$

… but the moment you start cubing-up any number greater than 1, the values rise rapidly:

$2^3 = 2 \times 2 \times 2 = 8$

$3^3 = 3 \times 3 \times 3 = 27$ (think of the Rubik Cube, which appears to consist of 'three lots of three-by-three', whichever way you slice it)

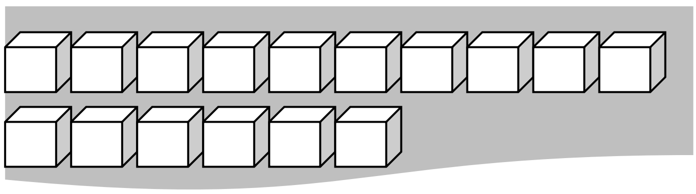
$4^3 = 4 \times 4 \times 4 = 64$

And by the time you reach 5^3 you are already into triple-digits at 125.

The sequence of Square Numbers went up more slowly (1, 4, 9, 16, 25 ...) since each term was only **raised to the power of two** rather than three.

Here's another quick illustration of how fast a cubic sequence rises.

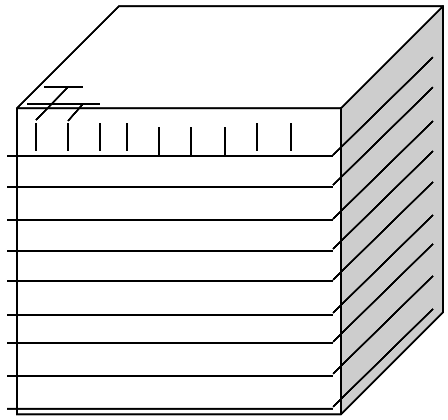
Imagine you are in a factory that makes 'widgets' (they could be sweets, ornaments, mechanical components ... for this purpose it scarcely matters). Each widget is in the form of a little cube, 1 cm^3. The widgets are produced in strips of 10, with 10 strips packaged on a square card:



The graphic shows a small quantity of widgets left on such a card: one whole strip of ten, plus six others. An entire card would have contained 10 rows of 10 = 10^2 = 100 widgets.

The cards are stacked and packed 10 to a carton, so the contents of an unopened carton are $10 \times 10 \times 10$ = 1,000 widgets (10 cards, each holding 10 rows of 10).

The cartons are packed onto a pallet, again in rows of 10; from front to back of the pallet, there are 10 rows, and these 10-by-10 layers of cartons are themselves stacked 10-high:



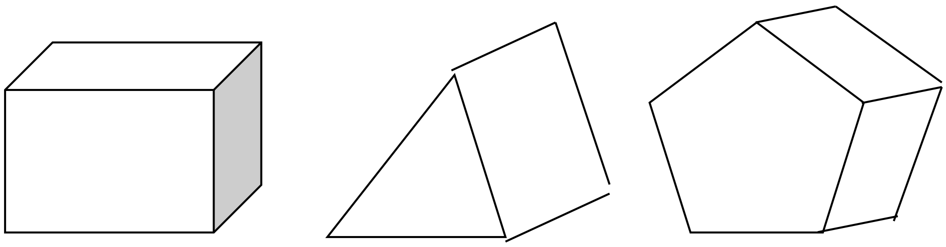
The pallet contains 10 × 10 × 10 = 1,000 cartons, each containing 10 × 10 × 10 = 1,000 widgets:

1,000 × 1,000 = 1,000,000 (one million widgets).

That's an awful lot of widgets!

Practical Calculations with Volume

It is easy enough to find the volume of any solid with a set of straight, parallel sides (what we call **prisms**, such as these:)



We simply calculate the area of the **cross-section** (*i.e.* of any identical 'slice') and then multiply that by how long, or deep, the shape is.

This picture (from *philipcoppens.com*) shows Rushton Triangular Lodge, a folly built in Northamptonshire in the mid-1590s. For various arcane reasons the design is riddled with the figure 3: not least that it has 3 sides, each of which is 33 feet long.

Ignoring such details as steps and windows, we can work out the triangular 'footprint' of its ground-plan:

$$\frac{\underline{33 \times 33}}{2}\ \text{ft}^2 = \frac{\underline{1089}}{2}\ \text{ft}^2 = 544.5\ \text{ft}^2$$

(Remember our formula for the area of triangles?)

If the walls are also 33ft high from ground to gutter, we can then calculate the total volume of the main bulk of the building, as follows:

$$544.5 \times 33\ \text{ft}^3 = \underline{17{,}968.5}\ \text{ft}^3$$

… in round terms, virtually 18 thousand cubic feet. (We could usefully have estimated this last stage by rounding it down as '500 × 30 = 15,000', and correctly expecting the actual answer to lie somewhere between that and 20,000.)

A more everyday example would be the volume of a packed sliced loaf in the shape of a **cuboid** (like the left-hand prism shown earlier: it has all-right-angled corners, like a cube, but not every set of its edges are of equal length, more like a shoe-box).

The loaf is 15 × 15 cm, giving each slice a spreadable surface area of 225 cm^2, and it is 25cm long, so its total volume is

$$15 \times 15 \times 25\ \text{cm} = 5625\ \text{cm}^3$$

You can work out the volume of any prism using this simple method, regardless of whether the shape is lying longwise or upright: sort out the cross-section first, then multiply by the 'through'-dimension.

It works just as well with a 'circular prism', which of course we know as a **cylinder**. If, instead of a loaf, you take a pack of round biscuits of radius 3.5cm ...

Surface area of the flat face of the biscuit = $\pi \times 3.5 \times 3.5$

$= \pi \times 12.25 \text{ cm}^2$

$= 38.48 \text{ cm}^2$

If the unopened barrel is 20cm long, the total volume of biscuit inside will be

$20 \times 38.48 = 769.6 \text{ cm}^3$

Don't eat them all at once!

Before you break for a well-earned snack, try at least some of the following Activity.

You should meanwhile also note that we have not yet covered the volumes of more complex solids such as those with tapering, non-parallel edges, nor the **cone** nor the **sphere** (= ball). Those will come at a later stage, but the kinds we have now met are very useful to be able to deal with in everyday life – as the Activity will show you.

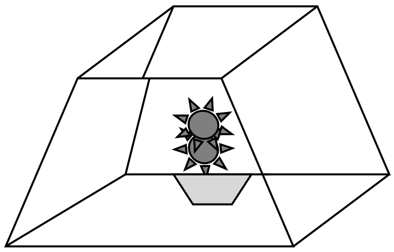
Activity 5	Now have a go at these problems involving volume ...

1. (a) A paperback book has pages 15cm by 10cm and the book is 2cm thick, cover-to-cover. What is the volume of the book?

   (b) This single book is the first in a 'saga' of five books. Assuming each book is more or less identical in format, what is the total volume of all five 'volumes' in the series?

2. A cylindrical bin stands 50cm tall and measures 30cm across its diameter. What is the capacity of the bin in cm^3?

3. Mr Gardiner wants to buy a modular greenhouse for his plants. The structure comes in kit form with 'ends' and 'middles', each with a common cross-section as shown:



The vertical height from ground to roof is 1.7 metres, the floor is 2 metres across and the width across the ceiling is 1.2 metres. Each section runs 1 metre 'deep' from front to back and comes with built-in supports, for joining together to make as long or short a tunnel as required.

(a) Work out

   i. the cross-section in square metres;
   ii. the volume (in cubic metres) of each section.

(b) Mr Gardiner reckons he needs between 10 and 12 m^3 for his plants.

   i. How many sections of greenhouse should he order?
   ii. How many 'middles' will he need, apart from the two ends?
   iii. What surface area of garden will be covered by the greenhouse?

4. A blow-up paddling pool has an internal diameter of 1.8 metres and a maximum depth of 70cm.

   (a) What would be the total volume of water if the pool were full to the brim?

   (b) In sensibly rounded terms, how much water would be needed to fill it to about two-thirds of the way up the side?

   (c) If a hose delivers water at 10,000 cm^3 (= 10 litres) per minute, about how long will it take for the pool to fill to the level you decided in part (b)?

5. A cylindrical tin can has a diameter of 6cm and is 11cm tall.

   (a) What is the *surface area* (**<u>not</u>** volume!) of a label wrapped round the can?

   (b) What is the surface area of the round lid of the can?

   (c) Ignoring any seams or overlaps, what it the total surface area of the can (*i.e.* both ends and round the outside)?

   (d) What is the volume of the tin?

Answers to Lesson Activities

Activity One

1. (a) P = 2 × (6 + 8) = 2 × 14 = 28 cm A = 6 × 8 = 48 cm^2
   (b) P = 2 × (7 + 13) = 2 × 20 = 40 cm A = 7 × 13 = 91 cm^2
   (c) P = 2 × (11 + 12) = 2 × 23 = 46 m A = 11 × 12 = 121 m^2
   (d) P = 2 × (18 + 24) = 2 × 42 = 84 mm A = 18 × 24 = 432 mm^2
   (e) P = 2 × (210 + 297) = 2 × 507 = 1014 mm; A = 210 × 297= 62370mm^2
   (f) P = 2 × (21 + 9) = 2 × 30 = 60 yib A = 21 × 9 = 189 sq. yib
   (g) P = 2 × (105 + 68) = 2 × 173 = 346m A = 105 × 68 = 7140 m^2

2. 2 × (3 + 1½) = 2 × 4½ = 9 metres

3. (a) The perimeter is 2 × (3 + 2) = 2 × 5 = 10 metres
   (b) 2 loops per cm along, @ 10cm per loop = 20cm of braid per cm of edge
   Therefore multiply edge length by 20, giving a total of **20 m** needed! (20m is about the length of a full-size cricket pitch, wicket to wicket, or of a 'long vehicle' such as a full-length passenger railway carriage or a traditional 'narrow-boat' – the longest of which are 70 feet.)

Activity Two

1. 50 × 50 = 2500; halve this to give 1250cm^2

2. Box area = 100 × 75 = 7500; kite area (half this) = 3750 cm^2

3. (a) 2.6 + 2.0 = 4.6; halve this to get the 'waistline' of 2.3m
   2.3 × 1.2 = 2.76 m^2
   (b) 2 × 2.76 = 5.52 m^2
   (c) 1.5 × 2.0 = 3 m^2
   (d) 1.3 × 1.5 = 1.95 m^2
   (e) 2 ends @ 1.95 = 3.90
   1 base @ 3.00
   2 sides @ 2.76 5.52
   12.42 m^2

4. (a) Surface area = 80 × 60 = 4800 cm^2
   (b) 4 sides'-worth @ 80cm = 320 cm.

Activity Four

1. Diameter 5m, so $\pi \times 5 = 15.7$ m

2. (a) Radius = 6378, so diameter (2 × radius) = 12756
   $12756 \times \pi = 40074$ km
   (b) 6357 × 2 = 12714
   $12714 \times \pi = 39942$ km
   Common-sense dictates that we regard 40,000 km as a workable compromise!

3. My trip round the circumference takes 150 seconds
   Walking the diameter would take me 'this long <u>divided by</u> *pi*' (which should obviously be shorter); taking a rough '*pi* = 3' we would estimate 50 seconds, *i.e.* rather less than a minute.
   150 divided by $\pi = 47.7$ seconds (say ¾ of a minute, in 'round terms', particularly if I happen to speed up a bit with the excitement of it all!)
   In this question we have been working not in units of length, but in 'lengths of time' taken to cover whatever those actual distances were. The principle is just the same.

4. The rope which defines the goat's 'working radius' is 4m long;
   $4 \times 4 = 16$; $16 \times \pi = 50\text{m}^2$ (to 2 significant figures; it happens to be a 'round number' again by happy coincidence)

5. The boom makes a working circumference of 2,500 metres, so its radius from the leak point should be

   '$2{,}500 \div \pi$' (to get back to the diameter)
   = 796m to the nearest whole metre

   (We might have estimated '2400 ÷ 3 = 800' which is pretty close)

   If we stick with the rounded figure of an 800m diameter – which may also recall our 'mug' research, with an 8cm diameter matching a 25cm circumference (to the nearest whole unit) – this gives us a radius of 400m. We were working with a radius of 4 units in Q.4 above, neatly enough; so we can quite easily establish that

   $400\text{ m}^2 \times 400\text{ m}^2 = 160{,}000\text{ m}^2$ (careful with all those noughts!)

   The contained slick area will be $\pi \times 160{,}000\text{ m}^2$ which, in an echo of Q.4 and with rather more zeros, comes out at almost exactly $500{,}000\text{ m}^2$: in layman's terms, half a million square metres.

   For interest, and put another way, this area would be 50 **hectares** (a hectare being an area 100m by 100m). We discovered earlier that a typical football pitch (at 7140 m^2) was about ¾ of a hectare, particularly by the time you include a working strip of spare space around the playing rectangle itself. So this oil-slick would be enough to engulf 4/3 × 50 football pitches, which is getting on for 70 of them. Imagine each stadium in the top three Divisions or so, just awash with oil. That's an awful lot of oil!

6. (a) The fanlight has a diameter of 90cm so its radius is 45cm. The total glazed area will be

$$\frac{\pi \times 45^2}{2}$$ (Halved, because there's not a complete circle).

$45^2 = 2025$; multiply this by π to give 6362 cm^2 (to nearest cm^2)

Half of this gives an area of <u>3181 cm^2</u> .

(b) We could start from a rounded figure of 3200 cm^2 or go down a bit to 3000 cm^2, either of which should **factorise** pretty tidily, *eg*

3200 = 80 × 40 (a 'double-square shape'; not v elegantly proportioned)
3200 = 50 × 64 (a much friendlier shape)

3000 = 50 × 60 (similar to the example just above)

Either of these last two would be pleasing and practical but you may well find workable alternatives of your own.

Activity Five

1. The volume is 15 × 10 × 2 which is 300 cm^3

2. The bin has a radius of 15cm so its surface area will be $\pi \times 15^2$ cm^2 which is 225 × π, or 707 cm^2 to the nearest whole centimetre. Multiply this by 50 for the depth, to give a total volume of 35,350 cm^3.

3. (a) (i) The parallel sides (floor & ceiling) are 2m and 1.2m which add to 3.2m, which we then halve to discover the measurement across the middle (= 1.6m). Multiply this by the vertical height of 1.7m to give 1.6 × 1.7 = 2.72 m^2

(ii) The volume of each section is 2.72 × 1 which is clearly 2.72 m^3

(b) (i) 4 × 2.72 = 10.88, which lies almost midway between 10 and 12, so he will want 4 sections …

(ii) 2 ends and 2 middles

(iii) The total floor will be 4m long by 2m wide = 8 m^2

4. (a) The diameter is 180cm so the radius is 90cm, squared to 8100 cm^2 and multiplied by π to give a surface area of 25,447 cm^2 (rounded to nearest cm). Multiply again by the depth of 70cm to give a volume of 1,781,290 cm^3 (*alias* just over one-and-three-quarter thousand litres, a litre being 10 × 10 × 10 cm)

(b) Two-thirds of an only-slightly-rounded 1800 litres is simple enough to establish as 1200 litres (since 18 ÷ 3 = 6, and 6 multiplied back up by 2 is 12)

(c) At 10 litres per minute it will take 120 minutes (= 1200 ÷ 10) which is 2 hours!

5. (a) The label, if peeled off, will be a rectangle: one side will be 11cm (up the tin), the length of other will be the same as circumference of the lid.

The lid has a diameter of 6cm: multiply this by π to give 18.85cm (to 2 decimal places), then again by 11 to reach a label area of 207.35 cm^2.

(b) The lid's surface area will be π × the square of its radius:

$\pi \times 3 \times 3 = 9\pi = 28.27$ cm^2 (to 2 d.p.)

(c) The total surface area is the sum of our answer to (a) plus double our answer to (b) – to allow for the top and bottom:
207.35 + 28.27 + 28.27 = 263.89 cm^2.

If you were being *immensely* pedantic you would double this number once again, to allow for the inside and outside; but in terms of flat sheet metal needed to form the can, the answer we reached is fine!

(d) The volume is the lid's cross-section × 11, = 311 cm^3 to the nearest cm.