Lesson Fourteen

Inequalities and Accuracy

Aims

The aim of this lesson is to enable you to:

- solve problems involving inequalities without resorting to graphical methods

Context

There are many areas of work in which inequalities appear. This is particularly true in the commercial world where there are several constraints on producing the end product. Inequalities will often be used in such situations in a technique which is known as linear programming. We do not study linear programming as such but many of the techniques studied form a basis for this important industrial application.

Oxford Open Learning

Inequalities

If you can spot an equation by the equals sign in the middle, you can identify an **inequality** by the fact that the equals sign has been replaced by some other sign.

Here are the four alternatives to the equals sign:

$>$ means 'greater than'
$<$ means 'less than'
$\geq$ means 'equal to or greater than'
$\leq$ means 'equal to or less than'

So an inequality is *not* the opposite of an equation. When the signs $\leq$ and $\geq$ are used, the quantity on the left of the sign *could be* equal to the quantity on the right.

Number lines may come in handy when dealing with inequalities so you may want to revise this topic first.

Rules for Solving Inequalities

There are two important points to be made here.

It is important to remember that the answers to problems involving inequalities will usually be a set of numbers rather than just one or two numbers when dealing with ordinary equations.

One of the more common mistakes when dealing with inequalities in this way is to multiply by a negative number and forget to change the direction of the inequality.

Solving inequalities is **almost** the same as solving equations. The few exceptions will be dealt with at the end. If you can solve a 'linear equation' such as $2x-1=11$, then you can also solve four related 'inequalities':

$2x-1=11$ $2x=11+1$ $2x=12$ $x=\frac{12}{2}$ $x=6$	$2x-1<11$ $2x<11+1$ $2x<12$ $x<\frac{12}{2}$ $x<6$	$2x-1\leq 11$ $2x\leq 11+1$ $2x\leq 12$ $x\leq\frac{12}{2}$ $x\leq 6$	$2x-1>11$ $2x>11+1$ $2x>12$ $x>\frac{12}{2}$ $x>6$	$2x-1\geq 11$ $2x\geq 11+1$ $2x\geq 12$ $x\geq\frac{12}{2}$ $x\geq 6$

The only difference between the columns is the sign. The left-hand column has an equals sign throughout. The second column is made from the first column by changing the 'equals' sign to the 'less than' sign: <. The third column has the 'less than or equal to' sign: $\le$. The fourth column has the 'greater than' sign: >. The fifth column has the 'greater than or equal to' sign: $\ge$.

Example 1

Solve the inequalities:

(a) $x+3<7$ **(b)** $5x>30$

(c) $\frac{x}{3}-2\le 4$ **(d)** $\frac{2(x+1)}{5}\ge 4$

Each answer also includes the solution for the corresponding **equation**.

	Equation	**Inequality**
(a)	$x+3=7$	$x+3<7$
	$x=7-3$	$x<7-3$
	$x=4$	$x<4$
(b)	$5x=30$	$5x>30$
	$x=\frac{30}{5}$	$x>\frac{30}{5}$
	$x=6$	$x>6$
(c)	$\frac{x}{3}-2=4$	$\frac{x}{3}-2\le 4$
	$\frac{x}{3}=4+2$	$\frac{x}{3}\le 4+2$
	$\frac{x}{3}=6$	$\frac{x}{3}\le 6$
	$x=6\times 3$	$x\le 6\times 3$
	$x=18$	$x\le 18$
(d)	$\frac{2(x+1)}{5}=4$	$\frac{2(x+1)}{5}\ge 4$
	$x+1=\frac{4\times 5}{2}$	$x+1\ge\frac{4\times 5}{2}$
	$x+1=10$	$x+1\ge 10$
	$x=10-1$	$x\ge 10-1$
	$x=9$	$x\ge 9$

The Exceptions

Consider the true statement $2 < 3$. One way of interpreting this statement is to say that two is to the left of three on the conventional number line.

What happens if we 'change the sign' of both sides of the inequality? This is equivalent to multiplying both sides by -1, and the result would be $-2 < -3$. However, this is not true! If you consult a number line, you will find that -2 is to the right of -3, and therefore $-2 > -3$. This leads to the rule:

When multiplying or dividing both sides of an inequality by a negative number, reverse the inequality.

'Reversing' the inequality means replacing $<$ by $>$ and *vice versa*, and replacing $\leq$ by $\geq$ and *vice versa*.

Another exception involves 'reciprocals'. Recall that 'reciprocal' means 'one over'. It is possible to take the reciprocal of both sides of an equation. Look what happens, however, if we 'take reciprocals' of both sides of the inequality $2 < 3$. It becomes $\frac{1}{2} < \frac{1}{3}$. This is false! One half is in fact bigger than one third. This leads to another rule:

When taking reciprocals of both sides of an inequality, reverse the inequality.

Example 2

When $p = 5$ and $q = 7$, the statement $p < q$ is true.

In which of the following cases is the statement $p < q$ also true?

(a) $p = \frac{1}{5}$ and $q = \frac{1}{7}$

(b) $p = -5$ and $q = -7$

(c) $p = -\frac{1}{5}$ and $q = -\frac{1}{7}$

(d) $p = 5^2$ and $q = 7^2$

(e) $p = \left(\frac{1}{5}\right)^2$ and $q = \left(\frac{1}{7}\right)^2$

Answer: (c) and (d)

Example 3

Solve the inequality $4x - 3 \geq 7x - 12$

There are two possible strategies: collect the x terms either on the left or the right-hand side.

If we collect the x terms on the right-hand side:

$-3 + 12 \geq 7x - 4x$
$9 \geq 3x$
$3 \geq x$

We would normally rewrite this inequality as $x \leq 3$.

If, instead, we decided to collect the x terms on the left-hand side:

$4x - 7x \geq -12 + 3$
$-3x \geq -9$
$x \leq \frac{-9}{-3}$
$x \leq 3$

Notice that when the -3 moves from the left to the right-hand side (equivalent to dividing both sides by -3), the inequality is reversed. Thus the first exception is quite important. The second method is harder since it requires the use of an "exception".

Whole Number Solutions?

On occasions we need to solve inequalities for whole numbers only. These are called the integer solutions.

Example 4

Find the whole numbers which satisfy the inequalities:
(a) $2 \leq x \leq 8$ **(b)** $5 \leq x < 7$ **(c)** $5 < x < 6$

(a) The required whole numbers are 2, 3, 4, 5, 6, 7, 8.
(b) The required whole numbers are 5, 6.
(c) There are no whole numbers in between.

Activity 1

Solve the following inequalities:

1. (a) $x - 5 \geq 6$ (b) $\frac{x}{7} \leq 6$ (c) $2(x - 3) < 12$

   (d) $\frac{x+2}{8} > 7$ (e) $4x + 3 \geq 27$ (f) $\frac{3x}{5} - 2 \leq 4$

2. (a) $9x + 1 < 2x + 22$ (b) $10x - 3 \geq 2x + 53$
   (c) $5x + 3 \leq 11x - 39$ (d) $3x - 4 > 12x - 67$

3. Find whole numbers which satisfy the following inequalities:
   (a) $-2 \leq x \leq 2$ (b) $9 < x \leq 10$ (c) $9 < x < 10$

Suggested Answers to Activities

Activity One

1. (a) $x \geq 11$ (b) $x \leq 42$ (c) $x < 9$
   (d) $x > 54$ (e) $x \geq 6$ (f) $x \leq 10$

2. (a) $x < 3$ (b) $x \geq 7$
   (c) $7 \leq x$ or $x \geq 7$ (d) $7 > x$ or $x < 7$

3. (a) -2, -1, 0, 1, 2 (b) 10 (c) There aren't any.

Tutor-Marked Assignment C

Work through these questions carefully and send the answers to your tutor.

1. Use the following values of p, q, r and s to find the values of the following expressions:

   $p = 3$, $q = 6$, $r = 7$ and $s = 0$.

   (a) $12 - r$

   (b) qr

   (c) q/p

   (d) s/pqr (4 marks)

2. Find the value of the letter in the following:

   (a) $x + 3 = 9$

   (b) $3y = 9$

   (c) $a/3 = 9$

   (d) $7 - e = 5$ (4 marks)

3. Simplify the following where possible:

   (a) $3x - x$

   (b) $5x + y$

   (c) $7x^2 - 4x^2$

   (d) $4p^2 + 3p - p$

   (e) $8xy^2 - 2x + 2xy^2$

   (f) $4ab^2 - 2a^2b + ab - 3a^2b$ (6 marks)

4. Remove the brackets from:

(a) $2(x - y)$

(b) $-5(2x - 3y)$

(c) $-(3x^2 - 2y)$

(d) $2x(3x - 5y)$

(e) $(5x + 3)(x - 4)$

(f) $(2x - 1)^2$ (6 marks)

5. Factorise the following:

(a) $xy + xz$ (1 mark)

(b) $4b^2 - 2b$ (2 marks)

(c) $3a^2b + 6a^2b + 9ab^2$ (3 marks)

6. Solve the following equations:

(a) $2x + 1 = 7$

(b) $2x - 9 = 25$

(c) $5x - 20 = 3x - 8$

(d) $4r - \frac{r}{4} = 15$

(8 marks)

7. The formula

$$s = ut + \tfrac{1}{2}gt^2$$

is used in calculations about motion under gravity.

Find s when $u = 5$, $t = 2$, $g = 10$. (2 marks)

8. The formula for the volume of a cone is

$$V = \tfrac{1}{3}\pi r^2 h$$

Find the value of h if we take $\pi = 3$, $r = 4$ and $v = 48$. (3 marks)

9. Make y the subject of:

   (a) $x + y = 8$

   (b) $4y - 2x = 17$

   (c) $\frac{2y}{3} - x = z$

(5 marks)

10. Einstein's famous formula states that $E = mc^2$.

    Rewrite the formula with c as the subject. (2 marks)

11. Use the trial and improvement method to solve the equation $2x^3 + 5 = 24$. Find a solution that is a positive number and give your answer correct to one decimal place. (4 marks)

12. Work out an algebraic expression for the nth term of the following sequences:

    (a) 8, 11, 14, 17, 20

    (b) 20, 15, 10, 5, 0 (2 marks)

13. One side of a rectangle is 2 cm longer than the other. The perimeter of the rectangle (that is, the distance all the way around the edge) is 24 cm. How long are the sides of the rectangle? (3 marks)

14. A fitness fanatic swims 40 lengths of his pool per day. After 5 years, he has swum 1825 km. How long is his swimming pool in metres? (Assume that there are 365 days per year.)

(3 marks)

15. Solve the following inequalities:

    (a) $x + 6 > 15$ (1 mark)

    (b) $7x < 42$ (1 mark)

    (c) $5x + 3 \geq 23$ (2 marks)

    (d) $4x + 5 < x + 20$ (2 marks)

(total: 6 marks)

TOTAL FOR TMA: 64 MARKS